Introduction to Improper Integrals

Integral is considered improper if:

- .
- .

Do: sketch $f(x) = \frac{1}{x}$ (include asymptotes) then Do: sketch $f(x) = \frac{1}{x^2}$

Recall limits involving infinity:

$\lim\limits_{x \to +\infty} \frac{1}{x} =$ $\lim\limits_{x \to -\infty} \frac{1}{x} =$ $\lim\limits_{x \to 0^+} \frac{1}{x^2} =$

Do: determine the following limits

Recall: indeterminate forms and L'Hospital's Rule

Explore areas under the curve:

ex. $\int_1^2 \frac{1}{x^2}\,dx =$

ex. $\int_1^3 \frac{1}{x^2}\,dx =$

Do: $\int_1^{10} \frac{1}{x^2}\,dx =$

Do: $\int_1^{100} \frac{1}{x^2}\,dx =$

Question: What happens to the area under the curve as the upper bound gets infinitely large?

ex. $\int_1^{\infty} \frac{1}{x^2}\,dx =$

Definition:

if then

or if then

this type of integral is ________________ if corresponding limit ___________

or it is _______________ if corresponding limit ____________________________.

assuming _______________________ are _____________________ then:

$\int_{-\infty}^{\infty} f(x)\, dx =$

ex. consider $\int_{1}^{\infty} \frac{1}{x}\, dx$

ex. $\int_{2}^{\infty} e^{-x} dx$

ex. $\int_{2}^{\infty} e^{x}\,dx$

ex. $\int_{-\infty}^{0} xe^{x}\,dx$